学籍番号　　　　　　　　　　　氏名

演習 [redacted] 8.1　次の式が成り立つことを示せ.

(1) $\nabla(\boldsymbol{a}\cdot\boldsymbol{b}) = \boldsymbol{a}\times(\nabla\times\boldsymbol{b}) + \boldsymbol{b}\times(\nabla\times\boldsymbol{a}) + (\boldsymbol{a}\cdot\nabla)\boldsymbol{b} + (\boldsymbol{b}\cdot\nabla)\boldsymbol{a}$

$\boldsymbol{a} = (a_1, a_2, a_3)$, $\boldsymbol{b} = (b_1, b_2, b_3)$ とおくと

$\nabla(\boldsymbol{a}\cdot\boldsymbol{b})$ の x 成分は $\frac{\partial}{\partial x}(a_1 b_1 + a_2 b_2 + a_3 b_3)$

$= \frac{\partial a_1}{\partial x} b_1 + a_1 \frac{\partial b_1}{\partial x} + \frac{\partial a_2}{\partial x} b_2 + a_2 \frac{\partial b_2}{\partial x} + \frac{\partial a_3}{\partial x} b_3 + a_3 \frac{\partial b_3}{\partial x}$

$\boldsymbol{a}\times(\nabla\times\boldsymbol{b})$ の x 成分は $a_2\left(\frac{\partial b_2}{\partial x} - \frac{\partial b_1}{\partial y}\right) - a_3\left(\frac{\partial b_1}{\partial z} - \frac{\partial b_3}{\partial x}\right)$

$\boldsymbol{b}\times(\nabla\times\boldsymbol{a})$ の x 成分は $b_2\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right) - b_3\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right)$

$(\boldsymbol{a}\cdot\nabla)\boldsymbol{b}$ の x 成分は $a_1 \frac{\partial b_1}{\partial x} + a_2 \frac{\partial b_1}{\partial y} + a_3 \frac{\partial b_1}{\partial z}$

$(\boldsymbol{b}\cdot\nabla)\boldsymbol{a}$ の x 成分は $b_1 \frac{\partial a_1}{\partial x} + b_2 \frac{\partial a_1}{\partial y} + b_3 \frac{\partial a_1}{\partial z}$

よって(右辺) = (左辺) が成り立つ

(2) $\nabla\times(\boldsymbol{a}\times\boldsymbol{b}) = (\boldsymbol{b}\cdot\nabla)\boldsymbol{a} - (\boldsymbol{a}\cdot\nabla)\boldsymbol{b} + (\nabla\cdot\boldsymbol{b})\boldsymbol{a} - (\nabla\cdot\boldsymbol{a})\boldsymbol{b}$

$\nabla\times(\boldsymbol{a}\times\boldsymbol{b})$ の x 成分は $\frac{\partial}{\partial y}(a_1 b_2 - a_2 b_1) - \frac{\partial}{\partial z}(a_3 b_1 - a_1 b_3)$

$= \frac{\partial a_1}{\partial y} b_2 + a_1 \frac{\partial b_2}{\partial y} - \frac{\partial a_2}{\partial y} b_1 - \boxed{a_2 \frac{\partial b_1}{\partial y}}$

$\quad - \frac{\partial a_3}{\partial z} b_1 - a_3 \frac{\partial b_1}{\partial z} + \frac{\partial a_1}{\partial z} b_3 + a_1 \frac{\partial b_3}{\partial z}$

$(\boldsymbol{b}\cdot\nabla)\boldsymbol{a}$ の x 成分は $b_1 \frac{\partial a_1}{\partial x} + b_2 \frac{\partial a_1}{\partial y} + b_3 \frac{\partial a_1}{\partial z}$

$-(\boldsymbol{a}\cdot\nabla)\boldsymbol{b}$ の x 成分は $-a_1 \frac{\partial b_1}{\partial x} - \boxed{a_2 \frac{\partial b_1}{\partial y}} - a_3 \frac{\partial b_1}{\partial z}$

$(\nabla\cdot\boldsymbol{b})\boldsymbol{a}$ の x 成分は $\left(\frac{\partial b_1}{\partial x} + \frac{\partial b_2}{\partial y} + \frac{\partial b_3}{\partial z}\right) a_1$

$-(\nabla\cdot\boldsymbol{a})\boldsymbol{b}$ の x 成分は $-\left(\frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z}\right) b_1$

よって(右辺) = (左辺) が成り立つ

演習 ~~9.2~~ 8.2 次の式が成り立つことを示せ（勾配，発散，回転に関する公式 I，II は使ってよい）.

(1) $\nabla \cdot (\nabla\phi \times \nabla\psi) = 0$

$$\nabla \cdot (\nabla\phi \times \nabla\psi) = \{\nabla \times (\nabla\phi)\} \cdot (\nabla\psi) - (\nabla\phi) \cdot \{\nabla \times (\nabla\psi)\} \quad (\because \mathrm{I}(5))$$

$$= \mathbf{0} \cdot (\nabla\psi) - (\nabla\phi) \cdot \mathbf{0} \quad (\because \mathrm{II}(1))$$

$$= 0$$

(2) $\nabla \times (\phi\nabla\psi) = \nabla\phi \times \nabla\psi$

$$\nabla \times (\phi\nabla\psi) = (\nabla\phi) \times (\nabla\psi) + \phi\nabla \times (\nabla\psi) \quad (\because \mathrm{I}(3))$$

$$= \nabla\phi \times \nabla\psi + \phi\,\mathbf{0} \quad (\because \mathrm{II}(1))$$

$$= \nabla\phi \times \nabla\psi$$

演習 ~~9.3~~ 8.3

$$\nabla \times (\nabla \times \boldsymbol{a}) = \nabla(\nabla \cdot \boldsymbol{a}) - \Delta\boldsymbol{a}$$

が成り立つことを示せ.

$\boldsymbol{a} = (a_1, a_2, a_3)$ とする

$\nabla \times (\nabla \times \boldsymbol{a})$ の x 成分は

$$\frac{\partial}{\partial y}\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right) - \frac{\partial}{\partial z}\left(\frac{\partial a_1}{\partial z} - \frac{\partial a_3}{\partial x}\right)$$

$$= \frac{\partial^2 a_2}{\partial x \partial y} - \frac{\partial^2 a_1}{\partial y^2} - \boxed{\frac{\partial^2 a_1}{\partial z^2}} + \frac{\partial^2 a_3}{\partial x \partial z}$$

$\nabla(\nabla \cdot \boldsymbol{a})$ の x 成分は $\dfrac{\partial}{\partial x}\left(\dfrac{\partial a_1}{\partial x} + \dfrac{\partial a_2}{\partial y} + \dfrac{\partial a_3}{\partial z}\right)$

$$= \cancel{\frac{\partial^2 a_1}{\partial x^2}} + \frac{\partial^2 a_2}{\partial x \partial y} + \frac{\partial^2 a_3}{\partial x \partial z}$$

$-\Delta\boldsymbol{a}$ の x 成分は $-\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}\right) a_1$

$$= -\cancel{\frac{\partial^2 a_1}{\partial x^2}} - \frac{\partial^2 a_1}{\partial y^2} - \boxed{\frac{\partial^2 a_1}{\partial z^2}}$$

よって (右辺) = (左辺) が成り立つ