

学籍番号

氏名

演習 2.1 次のラプラス変換を求めよ.

$$(1) \mathcal{L}[t \sin 3t]$$

$$= -\frac{d}{ds} \left(\frac{3}{s^2 + 9} \right)$$

$$= \frac{6s}{(s^2 + 9)^2}$$

$$(3) \mathcal{L} \left[\int_0^t t e^{-2t} dt \right]$$

$$= \frac{1}{s} \mathcal{L} [t e^{-2t}]$$

$$= \frac{1}{s} \frac{1}{(s+2)^2}$$

$$= \frac{1}{s(s+2)^2}$$

$$(2) \mathcal{L}[t \cos \sqrt{2}t]$$

$$= -\frac{d}{ds} \left(\frac{s}{s^2 + 2} \right)$$

$$= \frac{s^2 - 2}{(s^2 + 2)^2}$$

$$(4) \mathcal{L} \left[\int_0^t t \sin t dt \right]$$

$$= \frac{1}{s} \mathcal{L} [t \sin t]$$

$$= \frac{1}{s} \left(-\frac{d}{ds} \mathcal{L} \sin t \right)$$

$$= \frac{1}{s} \left(-\frac{d}{ds} \left(\frac{1}{s^2 + 1} \right) \right)$$

$$= \frac{1}{s} \cdot \frac{2s}{(s^2 + 1)^2}$$

$$= \frac{2}{(s^2 + 1)^2}$$

演習 2.2 次の関数 $F(s)$ のラプラス逆変換を求めよ.

$$(1) F(s) = \frac{1}{(s-3)^2}$$

$$\mathcal{L}^{-1} F(s) = t e^{3t}$$

$$(2) F(s) = \frac{s-5}{s^2+5}$$

$$= \frac{s}{s^2+5} - \frac{5}{s^2+5}$$

$$= \frac{s}{s^2+5} - \sqrt{5} \frac{\sqrt{5}}{s^2+5}$$

$$\mathcal{L}^{-1} F(s) = \cos \sqrt{5} t - \sqrt{5} \sin \sqrt{5} t$$

$$(3) F(s) = \frac{1}{s^3} + \frac{2}{s^2} - \frac{1}{3s}$$

$$= \frac{1}{s^3} + 2 \frac{1}{s^2} - \frac{1}{3} \cdot \frac{1}{s}$$

$$\mathcal{L}^{-1} F(s) = \frac{t^2}{2!} + 2t - \frac{1}{3}$$

$$= \frac{1}{2} t^2 + 2t - \frac{1}{3}$$

$$(4) F(s) = \frac{1}{(2s-3)^2}$$

$$= \frac{1}{4} \frac{1}{\left(s - \frac{3}{2}\right)^2}$$

$$\mathcal{L}^{-1} F(s) = \frac{1}{4} t e^{\frac{3}{2}t}$$

$$(5) F(s) = \frac{s+6}{s^2+9}$$

$$= \frac{s}{s^2+9} + \frac{6}{s^2+9}$$

$$= \frac{s}{s^2+9} + 2 \frac{3}{s^2+9}$$

$$\mathcal{L}^{-1} F(s) = \cos 3x + 2 \sin 3x$$

$$(6) F(s) = \frac{2s}{s^2+2s+5}$$

$$= \frac{2s}{(s+1)^2+4}$$

$$= \frac{2(s+1)}{(s+1)^2+4} - \frac{2}{(s+1)^2+4}$$

$$\mathcal{L}^{-1} F(s) = 2e^{-x} \cos 2x - e^{-x} \sin 2x$$

$$= e^{-x} (2 \cos 2x - \sin 2x)$$

$$(7) F(s) = \frac{2s-3}{s^2-5s+6}$$

$$= \frac{2s-3}{(s-2)(s-3)}$$

$$= \frac{a}{s-2} + \frac{b}{s-3} \quad \text{とあか}$$

$$a(s-3) + b(s-2) = 2s-3$$

$$\begin{cases} a+b=2 & a=-1 \\ -3a-2b=-3 & b=3 \end{cases}$$

$$F(s) = -\frac{1}{s-2} + 3 \frac{1}{s-3}$$

$$\mathcal{L}^{-1} F(s) = 3e^{3x} - e^{2x}$$

$$(8) F(s) = \frac{s}{(s-1)^2}$$

$$= \frac{a}{(s-1)^2} + \frac{b}{s-1} \quad \text{とあか}$$

$$a + b(s-1) = s$$

$$\begin{cases} b=1 & a=1 \\ a-b=0 & b=1 \end{cases}$$

$$F(s) = \frac{1}{(s-1)^2} + \frac{1}{s-1}$$

$$\mathcal{L}^{-1} F(s) = x e^x + e^x$$

$$= e^x (x+1)$$

$$(9) F(s) = \frac{s+3}{s(s^2+4)}$$

$$= \frac{a}{s} + \frac{bs+c}{s^2+4}$$

$$as^2+4a+bs^2+cs = s+3$$

$$\begin{cases} a+b=0 & a=\frac{3}{4} \\ c=1 & b=-\frac{3}{4} \\ 4a=3 & c=1 \end{cases}$$

$$F(s) = \frac{3}{4} \cdot \frac{1}{s} - \frac{3}{4} \frac{s}{s^2+4} + \frac{1}{s^2+4}$$

$$= \frac{3}{4} \cdot \frac{1}{s} - \frac{3}{4} \frac{s}{s^2+4} + \frac{1}{2} \frac{2}{s^2+4}$$

$$\mathcal{L}^{-1} F(s) = \frac{3}{4} - \frac{3}{4} \cos 2x + \frac{1}{2} \sin 2x$$

$$(10) F(s) = \frac{1}{s(s+2)^2}$$

$$= \frac{a}{s} + \frac{b}{(s+2)^2} + \frac{c}{s+2} \quad \text{とあか}$$

$$as^2+4as+4a+bs+cs^2+2cs = 1$$

$$\begin{cases} a+c=0 & a=\frac{1}{4} \\ 4a+b+2c=0 & b=-\frac{1}{2} \\ 4a=1 & c=-\frac{1}{4} \end{cases}$$

$$F(s) = \frac{1}{4} \cdot \frac{1}{s} - \frac{1}{2} \cdot \frac{1}{(s+2)^2} - \frac{1}{4} \frac{1}{s+2}$$

$$\mathcal{L}^{-1} F(s) = \frac{1}{4} - \frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x}$$

$$= \frac{1}{4} - \frac{1}{4} (2x e^{-2x} + e^{-2x})$$