

微分積分学II 期末テスト前演習問題(計算編) 解答

2018年1月18日

1.

1) 3

2) $2(\sqrt{2} - 1)$

3) 2

4) $\frac{1}{\sqrt{2}}$

5) $1 - \frac{1}{e^2}$

6) $\frac{\pi}{2}$

2.

1) $t = x^2 + x + 2$ とおくと、

$$\int_0^1 (2x+1)\sqrt{x^2+x+2} dx = \int_2^4 \sqrt{t} dt = \frac{4}{3}(4 - \sqrt{2}).$$

2) $t = \cos x$ とおくと、

$$\int_0^\pi (\cos^2 x + 1) \sin x dx = \int_1^{-1} -(t^2 + 1) dt = \frac{8}{3}.$$

3.

1)

$$\int_{-\pi}^\pi x \cos x dx = \int_{-\pi}^\pi x (\sin x)' dx = 0.$$

2)

$$\int_{-1}^1 x e^x dx = \int_{-1}^1 x (e^x)' dx = \frac{2}{e}.$$

4.

1) $x = -1$ のとき $\frac{1}{x+1}$ は定義されない。よって

$$\int_{-1}^0 \frac{1}{x+1} dx = \lim_{\varepsilon \rightarrow +0} \int_{-1+\varepsilon}^0 \frac{1}{x+1} dx = \lim_{\varepsilon \rightarrow +0} (0 - \log \varepsilon) = \infty.$$

2)

$$\int_2^\infty \frac{1}{x^4} dx = \lim_{K \rightarrow \infty} \int_2^K \frac{1}{x^4} dx = \lim_{K \rightarrow \infty} \left(-\frac{1}{3} \cdot K^{-3} + \frac{1}{3} \cdot 2^{-3} \right) = \frac{1}{24}.$$

3) $x = 1$ のとき $\frac{1}{\sqrt{1-x}}$ は定義されない。よって

$$\int_0^1 \frac{1}{\sqrt{1-x}} dx = \lim_{\varepsilon \rightarrow +0} \int_0^{1-\varepsilon} \frac{1}{\sqrt{1-x}} dx = \lim_{\varepsilon \rightarrow +0} (-2\sqrt{\varepsilon} + 2) = 2.$$

5.

1) $z_x = 3x^2y, z_y = x^3$

2) $z_x = 2ye^{2xy}, z_y = 2xe^{2xy}$

3) $z_x = 0, z_y = 2$

4) $z_x = 2(x+y), z_y = 2(x+y)$

5) $z_x = 2xy + 2, z_y = x^2$

6) $z_x = \cos(x+y^2),$

$z_y = 2y \cos(x+y^2)$

6.

1)

$$\frac{\partial z}{\partial x} = y, \frac{\partial z}{\partial y} = x + 2y, \frac{dx}{dt} = 2, \frac{dy}{dt} = -1$$

2)

$$\frac{dz}{dt} = -2t - 1$$

7.

1)

$$\frac{\partial z}{\partial x} = 2x + y, \frac{\partial z}{\partial y} = x + 2y$$

2)

$$\frac{\partial x}{\partial u} = 1, \frac{\partial x}{\partial v} = 1, \frac{\partial y}{\partial u} = 1, \frac{\partial y}{\partial v} = -1$$

3)

$$\frac{\partial z}{\partial u} = 6u, \frac{\partial z}{\partial v} = 2v$$