

微分積分学II 演習問題6 解答

2017年10月26日

1.

1) $t = \tan \frac{x}{2}$ と置くと、

$$\begin{aligned}\int \frac{1}{2 \sin x} dx &= \int \frac{1}{2} \cdot \frac{1}{\frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2} dt \\ &= \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \log|t| + C = \frac{1}{2} \log \left| \tan \frac{x}{2} \right| + C.\end{aligned}$$

2) $t = \sqrt{x-1}$ と置くと、 $t^2 = x-1 \rightsquigarrow x = t^2 + 1$. よって $dx = 2t dt$ であるから、

$$\begin{aligned}\int \frac{x^2}{\sqrt{x-1}} dx &= \int \frac{(t^2+1)^2}{t} \cdot 2t dt \\ &= 2 \int t^4 + 2t^2 + 1 dt \\ &= \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{4}{3}(x-1)^{\frac{3}{2}} + 2\sqrt{x-1} + C.\end{aligned}$$

2.

1) $t = \tan \frac{x}{2}$ と置くと、

$$\begin{aligned}\int \frac{1}{2 - 2 \sin x + \cos x} dx &= \int \frac{1}{2 - 2 \cdot \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2} dt \\ &= \int \frac{2}{2(1+t^2) - 2 \cdot 2t + (1-t^2)} dt \\ &= \int \frac{2}{t^2 - 4t + 3} dt \\ &= \int \frac{2}{(t-3)(t-1)} dt \\ &= \int \frac{1}{t-3} - \frac{1}{t-1} dt \\ &= \log|t-3| - \log|t-1| + C \\ &= \log \left| \tan \frac{x}{2} - 3 \right| - \log \left| \tan \frac{x}{2} - 1 \right| + C.\end{aligned}$$

2) $t = \sqrt{x+1}$ と置くと、 $t^2 = x+1 \rightsquigarrow x = t^2 - 1$. よって $dx = 2t dt$ であるから、

$$\begin{aligned} \int \frac{2}{x^2 \sqrt{x+1}} dx &= \int \frac{2}{(t^2-1)^2 t} \cdot 2t dt \\ &= \int \frac{4}{(t+1)^2 (t-1)^2} dt \\ &= \int \frac{1}{t+1} + \frac{1}{(t+1)^2} + \frac{1}{t-1} + \frac{1}{(t-1)^2} dt \\ &= \log|t+1| - \frac{1}{t+1} - \log|t-1| - \frac{1}{t-1} + C \\ &= \log \frac{|\sqrt{x+1}+1|}{|\sqrt{x+1}-1|} - \frac{1}{\sqrt{x+1}+1} - \frac{1}{\sqrt{x+1}-1} + C. \end{aligned}$$

3.

$$1) \left[\frac{1}{2} x^2 \right]_0^3 = \frac{9}{2} - 0 = \frac{9}{2}$$

$$2) \left[-\frac{1}{x} \right]_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$$

$$3) \left[e^x \right]_0^1 = e - 1$$

$$4) \left[\log|x| \right]_1^e = 1 - 0 = 1$$

$$5) \left[x^4 \right]_0^2 = 16 - 0 = 16$$

$$6) \left[\sin x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 1 - (-1) = 2$$

4. 教科書参照

5. 教科書参照