微分積分学II 演習問題4 解答

2017 年 10 月 12 日

1.

1)

$$\int x\cos x\,dx = \int x\,(\sin x)'\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C.$$

2)

$$\int 2xe^{2x}\,dx = \int x\left(e^{2x}\right)'\,dx = xe^{2x} - \int e^{2x}\,dx = \left(x - \frac{1}{2}\right)e^{2x} + C.$$

3)

$$I = \int \frac{\log x}{x}\,dx = \int \log x\,(\log x)'\,dx = (\log x)^2 - I.$$

よって

$$2I = (\log x)^2$$

$$\int \frac{\log x}{x}\,dx = \frac{(\log x)^2}{2} + C.$$

2.

1)

$$\int \log x^2\,dx = \int (x)'\log x^2\,dx = x\log x^2 - \int x\cdot\frac{2x}{x^2}\,dx = x\log x^2 - 2x + C.$$

2)

$$
\begin{aligned}
I = \int e^{\frac{1}{2}x} \cos 2x \, dx &= \int e^{\frac{1}{2}x} \left(\frac{1}{2} \sin 2x \right)' \, dx \\
&= \frac{1}{2} e^{\frac{1}{2}x} \sin 2x - \int \frac{1}{4} e^{\frac{1}{2}x} \sin 2x \, dx \\
&= \frac{1}{2} e^{\frac{1}{2}x} \sin 2x - \frac{1}{4} \int e^{\frac{1}{2}x} \left(-\frac{1}{2} \cos 2x \right)' \, dx \\
&= \frac{1}{2} e^{\frac{1}{2}x} \sin 2x - \frac{1}{4} \left(-\frac{1}{2} e^{\frac{1}{2}x} \cos 2x + \int \frac{1}{4} e^{\frac{1}{2}x} \cos 2x \, dx \right) \\
&= \frac{1}{2} e^{\frac{1}{2}x} \sin 2x + \frac{1}{8} e^{\frac{1}{2}x} \cos 2x - \frac{1}{16} I.
\end{aligned}
$$

よって

$$
\begin{aligned}
17I &= 2e^{\frac{1}{2}x} \left(4 \sin 2x + \cos 2x \right) \\
\int e^{\frac{1}{2}x} \cos 2x \, dx &= \frac{2}{17} e^{\frac{1}{2}x} \left(4 \sin 2x + \cos 2x \right) + C.
\end{aligned}
$$

3)

$$
\begin{aligned}
\int \tan^{-1} 2x \, dx &= \int (x)' \tan^{-1} 2x \, dx \\
&= x \tan^{-1} 2x - \int \frac{2x}{4x^2 + 1} \, dx
\end{aligned}
$$

$t = 4x^2 + 1$ と置くと、

$$
\begin{aligned}
&= x \tan^{-1} 2x - \frac{1}{4} \int \frac{1}{t} \, dt \\
&= x \tan^{-1} 2x - \frac{1}{4} \log \left(4x^2 + 1 \right) + C.
\end{aligned}
$$

4)

$$
\begin{aligned}
\int x \sin^{-1} x^2 \, dx &= \int \left(\frac{1}{2} x^2 \right)' \sin^{-1} x^2 \, dx \\
&= \frac{1}{2} x^2 \sin^{-1} x^2 - \int \frac{1}{2} x^2 \cdot \frac{2x}{\sqrt{1 - x^4}} \, dx
\end{aligned}
$$

$t = 1 - x^4$ と置くと、

$$
\begin{aligned}
&= \frac{1}{2} x^2 \sin^{-1} x^2 - \frac{1}{4} \int t^{-\frac{1}{2}} \, dt \\
&= \frac{1}{2} x^2 \sin^{-1} x^2 - \frac{1}{2} \sqrt{1 - x^4} + C.
\end{aligned}
$$

3.

1) $-\frac{2}{x}+\frac{2}{x-1}$　2) $\frac{1}{x}-\frac{2}{2x+1}$　3) $\frac{1}{x}+\frac{-x+2}{x^2+1}$
4) $\frac{1}{x}+\frac{x-1}{x^2+x+1}$　5) $-\frac{3}{x}+\frac{3}{x^2}+\frac{3}{x+1}$

4.

1) $-2\log|x|+\log|x-1|+C$

2) $\log|x|-\log|2x+1|+C$

3) $\frac{1}{x}+\frac{-x+2}{x^2+1}=\frac{1}{x}-\frac{1}{2}\cdot\frac{2x}{x^2+1}+2\cdot\frac{1}{x^2+1}$ より、$\log\frac{|x|}{\sqrt{x^2+1}}+2\tan^{-1}x+C.$

4) $\frac{1}{x}+\frac{x-1}{x^2+x+1}=\frac{1}{x}+\frac{1}{2}\cdot\frac{2x+1}{x^2+x+1}-\frac{3}{2}\cdot\frac{1}{x^2+x+1}=\frac{1}{x}+\frac{1}{2}\cdot\frac{2x+1}{x^2+x+1}-\frac{2}{3}\cdot\frac{1}{\left(\frac{2x+1}{\sqrt{3}}\right)^2+1}$ より、

$\log|x|+\frac{1}{2}\log|x^2+x+1|-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+C.$

5) $-3\log|x|-\frac{3}{x}+3\log|x+1|+C$

5. 教科書参照