

微分積分学 II 演習問題 3 解答

2017 年 10 月 5 日

1.

1) $t = x^3 + x + 3$ と置くと、

$$\int (6x^2 + 2)e^{x^3+x+3} dx = \int 2e^t dt = 2e^t + C = 2e^{x^3+x+3} + C.$$

2) $t = x^2$ と置くと、

$$\int x \sin x^2 dx = \int \frac{1}{2} \sin t dt = -\frac{1}{2} \cos t + C = -\frac{1}{2} \cos x^2 + C.$$

3) $t = x^2 + 4x + 1$ と置くと、

$$\int \frac{x+2}{x^2+4x+1} dx = \int \frac{1}{2t} dt = \frac{1}{2} \log |t| + C = \frac{1}{2} \log |x^2+4x+1| + C.$$

4) $t = x^3$ と置くと、

$$\int \frac{3x^2}{\sqrt{1-x^6}} dx = \int \frac{1}{\sqrt{1-t^2}} dt = \sin^{-1} t + C = \sin^{-1} x^3 + C.$$

2.

1) $t = \cos x$ と置くと、

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx = \int -\frac{1}{t} dt = -\log |\cos x| + C.$$

2) $t = x^2 + 1$ と置くと、

$$\int \frac{2x}{x^4+2x^2+2} dx = \int \frac{2x}{1+(x^2+1)^2} dx = \int \frac{1}{1+t^2} dt = \tan^{-1}(x^2+1) + C.$$

3) $t = \tan x$ と置くと、

$$\int \frac{1}{\sin x \cos x} dx = \int \frac{1}{\frac{\sin x}{\cos x} \cdot \cos^2 x} dx = \int \frac{1}{t} dt = \log |\tan x| + C.$$

4) $t = \frac{x+1}{2}$ と置くと、

$$\int \frac{1}{\sqrt{-x^2-2x+3}} dx = \int \frac{1}{2\sqrt{1-\left(\frac{x+1}{2}\right)^2}} dx = \int \frac{1}{\sqrt{1-t^2}} dt = \sin^{-1}\left(\frac{x+1}{2}\right) + C.$$

3.

1)

$$\int 3xe^{3x} dx = \int x (e^{3x})' dx = xe^{3x} - \int e^{3x} dx = xe^{3x} - \frac{1}{3}e^{3x} + C.$$

2)

$$\begin{aligned} \int 2x \sin x dx &= \int 2x (-\cos x)' dx \\ &= -2x \cos x + \int 2 \cos x dx = -2x \cos x + 2 \sin x + C. \end{aligned}$$

3)

$$\begin{aligned} \int 2x \log x dx &= \int (x^2)' \log x dx \\ &= x^2 \log x - \int x dx = x^2 \log x - \frac{1}{2}x^2 + C. \end{aligned}$$

4)

$$\begin{aligned} \int 20x (x+4)^3 dx &= \int 5x \{(x+4)^4\}' dx \\ &= 5x (x+4)^4 - \int 5 (x+4)^4 dx \\ &= 5x (x+4)^4 - (x+4)^5 + C. \end{aligned}$$

5)

$$\begin{aligned} I = \int e^x \cos x dx &= \int e^x (\sin x)' dx = e^x \sin x - \int e^x \sin x dx \\ &= e^x \sin x - \int e^x (-\cos x)' dx \\ &= e^x \sin x + e^x \cos x - I. \end{aligned}$$

よって

$$\begin{aligned} 2I &= e^x (\sin x + \cos x) \\ \int e^x \cos x dx &= \frac{e^x}{2} (\sin x + \cos x) + C. \end{aligned}$$

4. 教科書参照

5. 教科書参照