微分積分学II 演習問題12 解答

2018年1月12日

1.

1) $z_x = 1,\ z_y = 1$　　2) $z_x = y,\ z_y = x$　　3) $z_x = 2xy^3,\ z_y = 3x^2y^2$

4) $z_x = -y\sin(xy),\ z_y = -x\sin(xy)$

5) $z_x = -\dfrac{1}{2\sqrt{1-x+y}},\ z_y = \dfrac{1}{2\sqrt{1-x+y}}$

6) $z_x = y\cos(xy)\cos(x+y) - \sin(xy)\sin(x+y),$
$z_y = x\cos(xy)\cos(x+y) - \sin(xy)\sin(x+y)$

2.

1) $z = f(x,y) = x^2 + y^2$ とする。f を偏微分すると次の通り。

$$f_x(x,y) = 2x,\ f_y(x,y) = 2y$$

よって点 $(-1,2)$ での微分係数は

$$f_x(-1,2) = -2,\ f_y(-1,2) = 4.$$

したがって点 $(-1,2,5)$ における接平面は

$$\begin{aligned} z &= f_x(-1,2)\,(x+1) + f_y(-1,2)\,(y-2) + 5 \\ &= -2x + 4y - 5. \end{aligned}$$

2) $z = f(x,y) = x^3y$ とする。f を偏微分すると次の通り。

$$f_x(x,y) = 3x^2y,\ f_y(x,y) = x^3$$

よって点 $(1,-1)$ での微分係数は

$$f_x(1,-1) = -3,\ f_y(1,-1) = 1.$$

したがって点 $(1,-1,-1)$ における接平面は

$$\begin{aligned} z &= f_x(1,-1)\,(x-1) + f_y(1,-1)\,(y+1) - 1 \\ &= -3x + y + 3 \end{aligned}$$

3.

1)

$$\frac{\partial z}{\partial x} = y^2 = (t-1)^2,\ \frac{\partial z}{\partial y} = 2xy + 1 = 2t^2(t-1) + 1,\ \frac{dx}{dt} = 2t,\ \frac{dy}{dt} = 1$$

よって

$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} \\ &= 4t^3 - 6t^2 + 2t + 1.\end{aligned}$$

2)

$$\frac{\partial z}{\partial x} = 2x,\ \frac{\partial z}{\partial y} = -2y,\ \frac{dx}{dt} = \frac{e^t - e^{-t}}{2} = y,\ \frac{dy}{dt} = \frac{e^t + e^{-t}}{2} = x$$

よって

$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} \\ &= 0.\end{aligned}$$

4.

1)

$$\begin{aligned}&\frac{\partial z}{\partial x} = 1,\ \frac{\partial z}{\partial y} = -2y = -2(2u - v), \\ &\frac{\partial x}{\partial u} = -v,\ \frac{\partial y}{\partial u} = 2,\ \frac{\partial x}{\partial v} = -u,\ \frac{\partial y}{\partial v} = -1\end{aligned}$$

よって

$$\begin{aligned}\frac{\partial z}{\partial u} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u} \\ &= -8u + 3v, \\ \frac{\partial z}{\partial v} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v} \\ &= 3u - 2v.\end{aligned}$$

2)

$$\begin{aligned}&\frac{\partial z}{\partial x} = y^2 - 2x,\ \frac{\partial z}{\partial y} = 2xy, \\ &\frac{\partial x}{\partial u} = v,\ \frac{\partial y}{\partial u} = 1,\ \frac{\partial x}{\partial v} = u,\ \frac{\partial y}{\partial v} = 2\end{aligned}$$

よって

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\begin{aligned}
\frac{\partial z}{\partial u} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u} \\
&= 3u^2v + 6uv^2 + 4v^3, \\
\frac{\partial z}{\partial v} &= \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v} \\
&= u^3 + 6u^2v + 12uv^2.
\end{aligned}
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5. 教科書参照

6. 教科書参照